LECTURE NO 13

Electrostatics

Topics

- Electrostatics:
- Electrostatic fields,
- Coulombs law
- Electric field intensity

- Electrostatics is the study of Charges at Rest
- It is based on two fundamental law
- Coloumbs law
- Gauss Law

Coulomb's law states that the force F between two point charges Q_1 and Q_2 is:

1. Along the line joining them
2. Directly proportional to the product Q_1Q_2 of the charges
3. Inversely proportional to the square of the distance R between them.[3]

Expressed mathematically,

$$F = \frac{k\,Q_1 Q_2}{R^2} \tag{4.1}$$

where k is the proportionality constant. In SI units, charges Q_1 and Q_2 are in coulombs (C), the distance R is in meters (m), and the force F is in newtons (N) so that $k = 1/4\pi\varepsilon_o$. The constant ε_o is known as the *permittivity of free space* (in farads per meter) and has the value

$$\boxed{\begin{aligned} \varepsilon_o &= 8.854 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi}\ \text{F/m} \\ \text{or } k &= \frac{1}{4\pi\varepsilon_o} \simeq 9 \times 10^{9}\ \text{m/F} \end{aligned}} \tag{4.2}$$

Thus eq. (4.1) becomes

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2} \tag{4.3}$$

If point charges Q_1 and Q_2 are located at points having position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, then the force $\mathbf{F}_{12}$ on Q_2 due to Q_1, shown in Figure 4.1, is given by

$$\boxed{\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\,\mathbf{a}_{R_{12}}} \tag{4.4}$$

where

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 \tag{4.5a}$$

$$R = |\mathbf{R}_{12}| \tag{4.5b}$$

$$\mathbf{a}_{R_{12}} = \frac{\mathbf{R}_{12}}{R} \tag{4.5c}$$

By substituting eq. (4.5) into eq. (4.4), we may write eq. (4.4) as

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^3}\mathbf{R}_{12} \tag{4.6a}$$

or

$$\mathbf{F}_{12} = \frac{Q_1 Q_2 (\mathbf{r}_2 - \mathbf{r}_1)}{4\pi\varepsilon_0 |\mathbf{r}_2 - \mathbf{r}_1|^3} \tag{4.6b}$$

$$\mathbf{F} = \frac{QQ_1(\mathbf{r} - \mathbf{r}_1)}{4\pi\varepsilon_0|\mathbf{r} - \mathbf{r}_1|^3} + \frac{QQ_2(\mathbf{r} - \mathbf{r}_2)}{4\pi\varepsilon_0|\mathbf{r} - \mathbf{r}_2|^3} + \cdots + \frac{QQ_N(\mathbf{r} - \mathbf{r}_n)}{4\pi\varepsilon_0|\mathbf{r} - \mathbf{r}_N|^3}$$

$$\boxed{\mathbf{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}}$$